ENDOGENOUS FERTILITY WITH A SIBSHIP SIZE EFFECT

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Since the seminal work of Becker, the dynamics of endogenous fertility has been based on the trade-off faced by parents between the quantity and the quality of their children. However, in developing countries, where child labor is an indispensable source of household income, parents actually incur a negative cost by having an extra child, so that the trade-off disappears. The purpose of this paper is to restore the Beckerian quantity–quality trade-off when intergenerational transfers are upstream, in order to keep fertility endogenous. We do that by adding a negative "sibship size effect" on human capital formation to the standard Becker model. With a simple specification, we obtain multiplicity of steady states or, more fundamentally, the possibility of a jump from a state with high fertility and low income to a state with low fertility and high income, triggered by a continuous increase in the productivity of human capital formation.

Keywords: Endogenous Fertility, Intergenerational Transfers, Human Capital Formation, Demographic Transition

1. INTRODUCTION

In this paper, we propose an extension of the Beckerian model of endogenous fertility, in order to take into account the case of developing societies, where child labor is a necessity for family survival, and where net intergenerational transfers are upstream, from children to parents. Introducing a negative *sibship size effect* restores the quantity–quality trade-off at the basis of the Beckerian model, so that the endogenous character of the fertility rate as an interior solution to the parents' utility-maximization problem can be preserved.

Since Becker's seminal work, the modeling of endogenous fertility has indeed been based on the trade-off faced by parents between the quantity and the quality of their children.[1] This modeling has taken various formulations, from a simple

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version directly including the number of children in the parents' utility function to a rather sophisticated version in terms of a dynastic utility function, where the quality of each child, as assessed by the altruistic parent, is identified with the child's expected utility [see Becker and Barro (1988)].

Generally speaking, in all these models, the fertility decision is derived from the trade-off between two opposite effects of the number of children on the parent's utility. On one hand, as children are desirable, a larger number of them leads to higher utility. Also, each child's expected utility can be increased through bequests and education, adding to the utility of the altruistic parent. On the other hand, rearing children and investing in their expected utility is costly, implying a sacrifice in current consumption. As a consequence of this trade-off, the optimal number of children is typically an interior solution to the parent's problem. Taking child labor into account does not fundamentally change the nature of the trade-off, and therefore does not affect the analysis of fertility decisions, except that the wage earned by each child alleviates the family rearing cost.

The situation is, however, radically different as soon as child labor becomes so substantial that rearing costs, net of wage income, become negative. In this case, parents' decisions cease to be constrained by the trade-off between quantity and quality: increasing family size may actually allow the parents to improve the quality of their children through the higher household income afforded by child labor. In other words, when child labor is so substantial, the usual quantity–quality trade-off does not make it possible to find the optimal fertility decision, because there is in fact no more trade-off. In such a case, the desired number of children would attain its physiological upper bound.

The purpose of this paper is to extend the quantity–quality Beckerian model to the cases where net intergenerational income transfers are upstream, from children to parents. Notice that it is the possibility of these upstream income transfers, rather than the mere existence of child labor, that determines the need for this extension of the model of endogenous fertility.

Our paper shows that to restore the trade-off between quantity and quality, it is sufficient to add a *sibship size effect*. The sibship size effect, highlighted by the sociological and medical literature, condenses several negative effects of family size on the siblings' human capital formation and ultimately on family well-being. This literature, summarized in the following section, has indeed shown that the sibship size affects the siblings negatively, mostly through health and educational effects.

The main reason for generalizing the Beckerian model of endogenous fertility, and for allowing intergenerational transfers to go either downstream or upstream, is that both situations occur in the world economy. Clearly, in developed countries, transfers are from parents to children and, more fundamentally, child labor is rare. However, as we will recall in the next section, the assumption of transfers flowing from parents to children is not an accurate portrayal of some poor countries today, especially in Africa, where child labor is a necessity for family survival and intergenerational transfers are upstream. Nor does it portray what occurred

in Western Europe at the onset of industrialization, when the children of the proletariat, living in misery, used to start working as early as age four, generating a positive income for their families.[2]

Therefore, the aim of this paper is to develop a model that fits the family structure not only of developed countries but also of poor countries, in which child labor is a necessity. This model will have to allow intergenerational transfers to go either way, downstream and upstream. Thanks to the introduction of a sibship size effect, working in the process of capital formation, it will preserve the endogenous character of fertility as the result of parents' decisions, rather than downgrading fertility to an exogenous biological outcome. To allow a clearer comparison between the Beckerian model and our own, we shall make a minimal number of changes relative to Becker's canonical model.

The paper is divided into five sections. In Section 2 we introduce the sibship size effect and present facts on child labor and intergenerational transfers. Section 3 presents the model and discusses the significance of the sibship size effect. Section 4 analyzes the existence of steady state equilibria under different regimes of child labor and intergenerational transfers and suggests an approach to the demographic transition in terms of our extended model. Section 5 concludes.

2. RELATED LITERATURE AND EMPIRICAL REGULARITIES

We start this section with some evidence on the existence of a sibship size effect, before discussing the importance of child labor for poor families and of upstream intergenerational transfers for developing societies.

2.1. Effects of Family Size on Human Capital: The Sibship Size Effect

The medical and sociological literatures point out the negative influence of sibship size on the siblings' human capital formation, and more specifically on the level attained once a sibling has become an adult: the so-called *sibship size effect*. Two major components of this effect can be distinguished. The first, deteriorating health, is emphasized in the medical literature, whereas the second, delayed intellectual development, is mainly emphasized by the sociological literature.

The medical literature has shown the negative consequences for health of crowding and the consequent greater exposure to diseases. Infectious diseases are indeed more likely to occur in crowded households with numerous children. Some infections such as measles, chicken pox, and diarrhea are linked to family size [see Aaby et al. (1984) and Aaby (1988)]. As a consequence, larger families appear to increase the children's risk of contracting the infection and the severity of the infection among those who do become ill, and to lead to negative long-run effects on health and human capital. Moreover, Desai (1995) has shown that in poor countries, the addition of a sibling aged less than five years has a statistically negative impact on children's height for age-standardized core, which is a good proxy for children's global health.

The sociological and psychological literatures have analyzed the effects of family size on the emotive and intellectual development of children. Individual achievement tests in reading and mathematics confirm that an increasing number of siblings is associated with lower intellectual performance, even if the causal nature of this relationship has been contested [see Guo and VanWey (1999)]. Downey (2001) writes, "One of the most consistent predictors of educational outcomes is the number of siblings, or sibship size. Across various measures of intellectual skills and educational achievement, individuals with the fewest siblings do the best according to studies that have used multiple data sets collected in the United States, Europe, and Asia" (p. 497).[3]

Research in the economics of education is also available on this subject. For instance, according to Hanushek (1992), "the empirical analysis finds that achievement falls systematically with increased family size" [p. 112; see also Li et al. (2008)]. There are also empirical studies directly showing that the number of siblings adversely affects earned income [see Lampi and Nordblom (2012)]. Finally, this question has already been taken over in the economic literature devoted to the demographic transition. In a recent study, Klemp and Weisdorf (2012) use eighteenth to nineteenth century family reconstitution data from English parish records to "show that children of parents of low fecundity (and hence few siblings) were significantly more likely to become literate and find employment among the skilled professions than those of parents of high fecundity (and hence many siblings)" (p. 2).

Why do bigger families lead to lower human capital of their children? The literature has put forward three main reasons. The first has been suggested by the *resource dilution* theory, which claims that siblings compete for parents' finite time and attention, so that the fewer the better for their intellectual development and educational success.[4]

The second reason, stressed in particular by recent research on the situation of ultra-Orthodox Jewish women in Israel, England, and the United States, who have on the average more than seven children, is that these women are more often sick, and cannot take care of their children as well as healthy women [Taha et al. (2001); Strauss (2007); Wright et al. (2010)]. This effect results in educational deficiency for children in big families.

The third source of the sibship size effect, which lies on top of simple resource dilution, is scale diseconomies, as emphasized by the medical literature, concerning the risk of getting severe infections.

Thus, children in big families are ceteris paribus less developed intellectually and less healthy. Sibship size affects children's human capital negatively through different channels investigated by the sociological, psychological, medical, and economic literatures. The modeling of human capital formation should consequently take family size, and more precisely its negative effect, into account. This question has, however, been left principally to medical and sociological research, and there is certainly a need for further empirical economic research to assess the

strength of the sibship size effect, which we are tentatively assuming here to be signficant.

2.2. Intergenerational Transfers and the Contribution of Child Labor to Family Income

The International Labour Organization (ILO) 2010 report on child labor estimates at 215 million the number of children between the ages of 5 and 17 working full time, and at 306 million the number of children in the same class of age who are "doing some kind of work." These figures represent 13.6% and 19.3% of the whole world population in the same age range, respectively. Moreover, the ILO 2006 report indicated that 120 million children between the ages of 5 and 14, in the developing countries alone, were full-time workers.

The phenomenon of child labor is pervasive, but its intensity is higher in poorer countries. In a study covering the global economy, where child labor is presented as a symptom of poverty, Edmonds and Pavcnik (2005, p. 210, Fig. 1) show that there is a strong negative correlation between GDP per capita and economic activity rates for children. In particular, the importance of child income in alleviating household poverty varies over countries as shown in Table 1, which is based on ILO family surveys.

It is shown that in most of the reported cases half or more (up to 70%) of the families would see at least a reduction of living standards if children stopped working. Many families claim that without child labor, the household enterprise would stop operating, which would send them into poverty. In a study devoted to child labor in Nigeria, Okpukpara and Odurukwe (2006) report that "the contribution of children's earnings to household income ranges from 3.5% to 38%" (p. 25) according to the regions, and note that "many families have no alternative other than to send their children to work because they see their earnings as an input into family survival" (p. 27).[5]

Economic historians have shown that this phenomenon, observed nowadays in poor countries, was also prominent in England at the time of the industrialization. Indeed, child labor amounted in the nineteenth century to a significant part of the workforce in some British industries. Children under 12 years of age constituted 8% of the labor force in the cotton industry, and children aged between 13 and 18 another 10% [see Evans (1990, p. 250)]. In the 1830s, in some regions such as Lancashire and Leeds, 36% of the workforce in the textile industry consisted of children under the age of 16 [see Tuttle and Wegge (2002)]. As stressed by Horrell and Humphries (1997), "the contributions of women and children may have been crucial to most families during certain stages in the family life cycle" (p. 35), and "in only a few occupations were men earning enough to buy their families sustenance and to provide the roof over their heads; for most households the earnings of women and children were essential" (p. 42). Focusing on the working class, Schellekens (1993, p. 3) writes, "men's wages among the working class, and among unskilled laborers in particular, were not sufficient to support a

TABLE 1. Consequences to household if working children stopped work

	Attending school					Not attending school				
	Household living standard declines	Household cannot afford to live	Household enterprise cannot operate	Has no effects	Other	Household living standard declines	Household cannot afford to live	Household enterprise cannot operate	Has no effects	Other
Nigeria	30.7	2.1	19.0	N/A	48.3	23.9	1.7	22.1	N/A	52.3
Sri Lanka	16.3	1.0	31.1	46.9	4.8	45.8	3.6	15.6	30.2	4.8
Zimbabwe[a]	33.1	7.7	19.0	36.5	3.7	—	—	—	—	—
Ghana[a]	43.8	4.9	21.6	28.9	0.6	—	—	—	—	—
Malawi	N/A	N/A	N/A	N/A	N/A	15.5	5.0	7.2	25.1	47.2

[a] Including both attending and not attending school.
Source: ILO Surveys on Child Labor (2006).

family." Moreover, according to Shammas (1984), adult equivalent caloric intakes were only just at minimum subsistence levels in the 1790s. Because real earnings of men fell until the 1830s, an increase in child labor was a necessity to keep people alive, and out of complete misery.

In conclusion, there are periods in history, countries, and regions in which intergenerational transfers are from children to parents. The model we are going to present in the next section takes this fact into account and illustrates the role of the sibship size effect in making upstream intergenerational transfers compatible with the theory of endogenous fertility.[6]

3. THE MODEL

We analyze an economy in which intrahousehold transfers can indifferently take place from parents to children or from children to parents. Let such an economy have overlapping generations, each with a continuum of identical households. Households are composed of individuals living for two periods, childhood and adulthood. Household consumption is not individualized: it covers consumption by adults and children. A child born at period t participates in household consumption c_t, is reared at a fixed cost σ_t, works during $l_t \in [0, 1]$ units of time at a wage rate w_t, and is educated at no extra cost during the remaining time. Education of a child belonging to a sibship of size n_t during $1 - l_t$ units of time leads in adulthood to human capital generating a number $H(l_t, n_t)$ of efficiency units per unit of working time. The function H of human capital formation is assumed to be continuously differentiable, decreasing, and concave. H is decreasing in l_t because more working time l_t means less spare time left for education, and it is decreasing in n_t because of the assumed negative *sibship size effect* on human capital formation.

3.1. The Parent's Decisions

Household decisions concerning the children of generation t are assumed to be taken by a representative adult of generation $t - 1$, the parent. They address the number $n_t \in \mathbf{R}_+$ of children, their individual labor supply $l_t \in [0, 1]$, and the bequest $b_t \in \mathbf{R}_+$ to be left to each child. For simplicity, we assume a constant environment. This means, in particular, constancy of the wage per efficiency unit ($w_t = w_{t-1} = w$), implicitly resulting from output production by competitive firms endowed with a linear technology. It also means constancy of the cost of rearing a child, assumed smaller than the wage ($\sigma_t = \sigma_{t-1} = \sigma < w$), in order for intergenerational transfers from children to parents, through child labor, to be possible. Given adult income y_t at period t and a degree of altruism $\delta n_t^{-\varepsilon}$ (with $0 < \delta < 1$ and $\varepsilon \geq 0$) toward each child [Becker and Barro (1988); Barro and

Becker (1989)], household decisions are consequently taken to solve the program

$$V(y_t) \equiv \max_{(n_t, l_t, b_t)} \left\{ \begin{array}{c} U\left[y_t - (\sigma - wl_t + b_t)\, n_t\right] + \left(\delta n_t^{-\varepsilon}\right) n_t\, V\,(y_{t+1})| \\ y_{t+1} = w\,H\,(l_t, n_t) + b_t \end{array} \right\}. \quad \text{(1)}$$

The function U represents current utility, which depends exclusively on household consumption $c_t = y_t - (\sigma_t - w_t l_t + b_t)\, n_t$ and is assumed to be continuously differentiable, increasing, and strictly concave.

The value function V represents the maximum utility an adult can obtain from each given income, including, in addition to current utility $U(c_t)$ derived from household consumption, the sum $n_t\left(\delta n_t^{-\varepsilon} V(y_{t+1})\right)$ of values $V(y_{t+1})$ of all (identical) children, weighted by the degree of altruism $\delta n_t^{-\varepsilon}$ toward each of them. By induction, we see that this recursive formulation is equivalent to a dynastic formulation of the type introduced by Becker and Barro (1988), namely

$$V(y_0) = \max_{(n_t, l_t, b_t)_{t \in \mathbb{N}}} \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \delta^t N_t^{1-\varepsilon} U(c_t)| \\ N_0 = 1,\, c_0 = y_0 - (\sigma - wl_0 + b_0)\, n_0 \\ \text{and, for } t \geq 1,\, N_t = \Pi_{i=0}^{t-1} n_i, \\ c_t = w\,H\,(l_{t-1}, n_{t-1}) + b_{t-1} - (\sigma - wl_t + b_t)\, n_t \end{array} \right\}, \quad \text{(2)}$$

provided the value of the objective function, an infinite sum, *remains finite*.

Barro and Becker (1989) assume in general that the current utility function is isoelastic ($U(c) = (1/\alpha)\, c^{\alpha}$, with $\alpha < 1$, $\alpha \neq 0$); in other words, that the elasticity of intertemporal substitution is constant, equal to $1/(1 - \alpha) > 0$. This specification covers two cases: the case of *intertemporal substitutability*, with $\alpha > 0$, and the case of *intertemporal complementarity*, with $\alpha < 0$.

Because the utility derived from the sibship born at t is then equal to $N_t^{1-\varepsilon}(1/\alpha)\, c_t^{\alpha}$, we must assume, in order to express the idea that parents like having children, that $1 - \varepsilon$ and α have the same sign, that is, $\varepsilon < 1$ if $\alpha > 0$ and $\varepsilon > 1$ if $\alpha < 0$ [Jones and Schoonbroodt (2010)].

If we refer to the recursive program (1), we see that the first-order condition for utility maximization relative to a positive number n_t of children can be written as the equality of the marginal opportunity cost and the marginal benefit of having an extra child:

$$U'(c_t)\,(\sigma - wl_t + b_t)$$
$$= \delta\,(1 - \varepsilon)\, n_t^{-\varepsilon} V(y_{t+1}) + \delta n_t^{1-\varepsilon} V'(y_{t+1})\, w\,H_n'(l_t, n_t). \quad \text{(3)}$$

By denoting as $\epsilon_V(y_{t+1}) = \left|V'(y_{t+1})\, y_{t+1}/V(y_{t+1})\right|$ the elasticity (in absolute value) of V at y_{t+1} and as $\epsilon_{Hn}(l_t, n_t) = \left|H_n'(l_t, n_t)\, n_t/H(l_t, n_t)\right|$ the elasticity (in absolute value) of H with respect to n at (l_t, n_t), we can rewrite this first-order condition as

$$U'(c_t)\,(\sigma - wl_t + b_t)$$
$$= \delta n_t^{-\varepsilon}\left|V(y_{t+1})\right|\left(|1 - \varepsilon| - (1 - b_t/y_{t+1})\,\epsilon_V(y_{t+1})\,\epsilon_{Hn}(l_t, n_t)\right). \quad \text{(4)}$$

Notice that the right-hand side (RHS) of this equation (the marginal utility of an extra child) still covers both the case of intertemporal substitutability, with $V(y_{t+1}) > 0$ and $1 - \varepsilon > 0$, and the case of intertemporal complementarity, with $V(y_{t+1}) < 0$ and $1 - \varepsilon < 0$.[7]

3.2. Intergenerational Transfers

By the first-order condition (4), in spite of child labor ($l_t > 0$), utility-maximizing intergenerational transfers are necessarily downstream, from parents to children ($\sigma + b_t > wl_t$), as long as the negative *sibship size effect* on human capital formation (measured by the elasticity $\epsilon_{Hn}(l_t, n_t)$) is kept small enough, that is, as long as $\epsilon_{Hn}(l_t, n_t) < |1 - \varepsilon| / ((1 - b_t/y_{t+1}) \epsilon_V(y_{t+1}))$.

Upstream intergenerational transfers correspond to a negative marginal opportunity cost of an extra child, so that the number of children is pushed to its biological upper bound unless the marginal utility of an extra child, diminished by a strong sibship size effect, itself becomes negative. We formally state this result:

PROPOSITION 1. *A solution* (n_t, l_t, b_t) *to program (1) can exhibit upstream intergenerational transfers, from children to parents* ($\sigma + b_t < wl_t$), *only under a strong enough sibship size effect, as measured by the elasticity (in absolute value) of the function H with respect to n:*

$$\epsilon_{Hn}(l_t, n_t) > \frac{|1 - \varepsilon|}{(1 - b_t/y_{t+1}) \epsilon_V(y_{t+1})}. \tag{5}$$

A further point should be emphasized at this stage. The two regimes of intergenerational transfers may rule in two different economies, but they may also be alternatively viable in the same economy (with unchanged specifications) if the sibship size effect varies in intensity with (l, n).

If, for instance, the elasticity $\epsilon_{Hn}(l, n)$ is an increasing function of n, we may well obtain existence of two contrasting steady state equilibrium regimes: one with low fertility, a small sibship size effect, and downstream intergenerational transfers, and another with high fertility, a large sibship size effect, and upstream intergenerational transfers. We illustrate this possibility in the following section.

Before proceeding to the analysis of steady states, it may be useful to compare these preliminary conclusions with what we find in the related literature.[8] A negative sibship size effect is in fact already present, at least implicitly and in a mild form, in Becker et al. (1990), through the assumption that each child's human capital is a function of the *parent's* time invested in her/his education. As time availability is limited, this leads to *resource dilution*, one of the possible sources of a negative sibship size effect.[9] This effect is responsible for the possible coexistence of two steady states, one (corresponding to a "poverty trap") with high fertility and no investment in human capital, the other with low fertility and possibly endogenous growth (because of human capital accumulation from one

generation to the next, which is ignored in our model). Notice that, as children do not work, intergenerational transfers remain downstream.

The same source of a sibship size effect, namely dilution of the parent's time endowment, has been used again and again in a series of models with child labor and intergenerational transfers going either way, in order to obtain existence of interior, possibly multiple, steady states:[10] Dessy (2000), Galor and Weil (2000), Hazan and Berdugo (2002), Wigniolle (2002), Blackburn and Cipriani (2005), Chakraborty and Das (2005), Sugawara (2010), and Varvarigos and Zakaria (2013). Should we eliminate resource dilution in these models, we would obtain a single regime of extreme upstream intergenerational transfers, corresponding in our framework to a corner solution with $l = 1$ and n equal to some exogenous upper bound $\bar{n}$ that we did not explicitly introduce.[11]

4. STEADY STATES

From now on, we assume isoelasticity of the current utility function $U(c) = (1/\alpha) c^{\alpha}$, with $\alpha < 1$, $\alpha \neq 0$, and $(1 - \varepsilon)/\alpha \geq 1$ (for a degree of altruism $\delta n^{-\varepsilon}$, $\varepsilon > 0$).[12] For simplicity, we further assume that the function describing human capital formation is multiplicatively separable and linear affine in both its arguments, $l \in [0, 1]$ and $n \in [0, \nu]$:

$$H(l, n) = h(1 - l/\lambda)(1 - n/\nu), \tag{6}$$

with a productivity parameter $h > 1$ and two parameters $\lambda > 1$ and $\nu > 2$ controlling the sensitivity of H to variations in its two arguments, l and n. This specification implies in particular that the sibship size effect $\epsilon_{Hn}(l, n) = n/(\nu - n)$ is an increasing function of n.

We look for steady state equilibria, along which all the variables are stationary. As the value of the objective function in the dynastic program (2) must remain finite, we have to impose the condition $\delta n^{1-\varepsilon} < 1$ on any admissible steady state value n.

4.1. Implications of the First-Order Conditions

We begin our analysis with first-order conditions relative to the three decision variables b_t, l_t, and n_t.

The bequest. Referring to the dynastic program (2), we see that the condition relative to the bequest b_t (for any $t \geq 0$) can be expressed as

$$\delta^t N_{t+1}^{1-\varepsilon} \left(\delta c_{t+1}^{\alpha-1} - c_t^{\alpha-1} n_t^{\varepsilon} \right) \leq 0, \tag{7}$$

with equality if $b_t > 0$. It is easy to check that the corresponding second-order condition is satisfied [the left-hand side (LHS) of this inequality is decreasing in

b_t]. Inequality (7) implies that

$$\frac{n_t^\varepsilon}{\delta} \left(\frac{c_{t+1}}{c_t}\right)^{1-\alpha} \geq 1, \tag{8}$$

so that $\delta n^{-\varepsilon} \leq 1$ in a steady state equilibrium, a condition that is satisfied *strictly* as soon as $n \geq 1$. Thus, *bequests are always zero* in the steady state equilibria we are going to consider, in which the representative family has at least one child.

Labor time. The first-order condition relative to l_t (for any $t \geq 0$) can be expressed as

$$-\delta^t N_{t+1}^{1-\varepsilon} w \left(\delta c_{t+1}^{\alpha-1} (h/\lambda) (1 - n_t/\nu) - c_t^{\alpha-1} n_t^\varepsilon\right) = 0 \text{ for } l_t \in (0, 1),$$

$$\leq 0 \text{ if } l_t = 0,$$

$$\geq 0 \text{ if } l_t = 1. \tag{9}$$

It is easily checked that the second-order condition is again satisfied in this case [the LHS of (9) is decreasing in l_t]. The preceding equation applying to an interior steady state solution $l \in (0, 1)$ determines the unique value $\widehat{n}$ compatible with such interior solution. It is such that

$$\delta \widehat{n}^{1-\varepsilon} (1/\widehat{n} - 1/\nu) = \lambda/h. \tag{10}$$

As the LHS of this equation is decreasing in n, tending to ∞ as $n \to 0$ and to 0 as $n \to \nu$, the value $\widehat{n} \in (0, \nu)$ is uniquely determined, and decreasing with λ/h. Moreover, $\widehat{n} > 1$ if $\lambda/h < \delta (1 - 1/\nu)$. Finally, by (9), $l = 0$ for $n < \widehat{n}$ and $l = 1$ for $n > \widehat{n}$: *full schooling* is thus associated with low fertility, *full child labor* with high fertility.

Family size. Finally, the first-order conditions relative to $n_0, \ldots, n_t, \ldots$ will be expressed in terms of $N_1, \ldots, N_{t+1}, \ldots$ by taking $n_t = N_{t+1}/N_t$, in order to get rid of the infinite sum in (2). The condition relative to N_t (for $t \geq 1$) is then

$$((1 - \varepsilon)/\alpha) \delta^t N_t^{-\varepsilon} c_t^\alpha$$
$$+ \delta^t N_t^{1-\varepsilon} c_t^{\alpha-1} \left((\sigma - w l_t + b_t) N_{t+1} N_t^{-2} - w (h/\nu) (1 - l_{t-1}/\lambda) N_{t-1}^{-1}\right)$$
$$+ \delta^{t+1} N_{t+1}^{1-\varepsilon} c_{t+1}^{\alpha-1} w (h/\nu) (1 - l_t/\lambda) N_{t+1} N_t^{-2}$$
$$- \delta^{t-1} N_{t-1}^{1-\varepsilon} c_{t-1}^{\alpha-1} (\sigma - w l_{t-1} + b_{t-1}) N_{t-1}^{-1} = 0, \tag{11}$$

the corresponding second-order condition being satisfied under the assumption $(1 - \varepsilon)/\alpha \geq 1$ (see the Appendix). For steady state equilibrium values $N_t = n^t$, $l_t = l$, and $b_t = 0$, we obtain

$$hF(n) \equiv h\delta n^{1-\varepsilon} \frac{(1/n - 1/\nu)(1 - \varepsilon)/\alpha - (1/\nu)\left(1 - \delta n^{1-\varepsilon}\right)}{\delta n^{1-\varepsilon}(1 - \varepsilon)/\alpha + 1 - \delta n^{1-\varepsilon}} = \frac{\sigma/w - l}{1 - l/\lambda} \equiv G(l), \tag{12}$$

expressing the equality of the benefit (on the LHS) and the real net cost (on the RHS) of bringing to maturity an extra child who works l time units.

Observe that we must restrict the admissible values of n to keep $\delta n^{1-\varepsilon}$ smaller than 1 (for the objective function of the dynastic program to remain finite): $n \in \left[0, \delta^{-1/(1-\varepsilon)}\right)$ if $\varepsilon < 1$ and $\left(\delta^{-1/(1-\varepsilon)}, \nu\right]$ if $\varepsilon > 1$. The denominator of the fraction on the LHS of equation (12) is then always positive. The derivative of the numerator with respect to n is

$$\frac{1-\varepsilon}{\alpha \nu n}\left(\alpha \delta n^{1-\varepsilon} - \frac{\nu}{n}\right) < 0, \tag{13}$$

so that the function F can change its sign at most once, possibly becoming negative for higher values of n. Looking at this numerator in (12), we see that, as n tends to ν, the first term, $(1/n - 1/\nu)(1-\varepsilon)/\alpha$, tends to zero, whereas the second, $-(1/\nu)\left(1 - \delta n^{1-\varepsilon}\right)$, remains negative if $\delta \nu^{1-\varepsilon} < 1$. Thus, in the case of intertemporal complementarity (where $\varepsilon > 1$), as well as in the case of intertemporal substitutability (where $\varepsilon < 1$) under this explicit condition,[13] $F(n)$ eventually becomes negative as n increases toward ν, when

$$\epsilon_{Hn}(l, n) = \frac{n}{\nu - n} > \frac{(1-\varepsilon)/\alpha}{1 - \delta n^{1-\varepsilon}}, \tag{14}$$

that is, when the sibship size effect as measured by the elasticity in absolute value of H with respect to n becomes large enough. Steady state intergenerational transfers then become upstream ($\sigma < wl$).

Notice that for $\varepsilon < 1$, if $\delta \nu^{1-\varepsilon} \geq 1$, $F\left(\delta^{-1/(1-\varepsilon)}\right) > 0$, so that $F(n)$ is always positive, excluding the possibility of a steady state with $l = 1$, and more generally excluding the possibility of transfers from children to parents [which imply that $G(l) < 0$]. We thus find the result stated in Proposition 1.

4.2. The Regimes of Steady State Equilibrium

As stated in the following proposition, which builds on our preceding analysis, three regimes of steady state equilibrium are possible:

PROPOSITION 2. *Let $\widehat{n}$ be the family size that is compatible with an interior steady state solution $l \in (0, 1)$, namely the value of n satisfying equation (10). A steady state equilibrium belongs to one of three possible regimes: (i) the regime of full child labor ($l = 1$), with high fertility [$\widehat{n} \leq n < \nu$, n s.t. $hF(n) = G(1)$], a strong sibship size effect, and transfers from children to parents; (ii) the regime of full schooling ($l = 0$), with low fertility [$0 < n \leq \widehat{n}$, n s.t. $hF(n) = G(0)$], a weak sibship size effect, and transfers from parents to children; (iii) the intermediate regime [$n = \widehat{n}$, $l \in (0, 1)$ s.t. $hF(\widehat{n}) = G(l)$], where intergenerational transfers may a priori go either way. Upstream intergenerational transfers are excluded in the case of intertemporal substitutability ($\alpha > 0$ and $\varepsilon < 1$) if $\delta \nu^{1-\varepsilon} \geq 1$.*

These three regimes can alternatively characterize a unique steady state, but they can also coexist in the same economy. By the first-order condition (9) relative
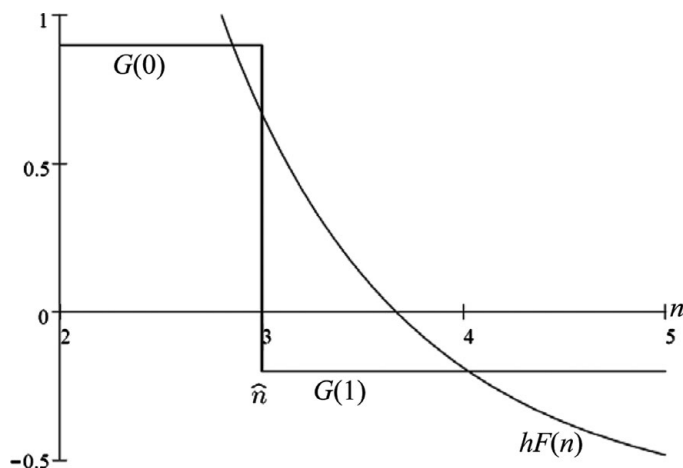
FIGURE 1. Multiplicity of steady states.

to l, the dependence on n of the RHS of equation (12) can be represented by a decreasing staircase curve, with an upper step $G(0) > 0$ for $n < \widehat{n}$, a lower step $G(1) < 0$ for $n > \widehat{n}$, and a vertical segment linking the two steps at $n = \widehat{n}$. A steady state equilibrium is determined by the intersection of this cost curve with the graph of $hF(n)$, representing the LHS of equation (12) and reflecting utility. This is illustrated in Figure 1, in a case where the three steady state regimes coexist.[14]

Of course, the existence of three steady states will not necessarily survive perturbations of the parameter values. However, by continuity of the functions F and G, existence is obtained through appropriate boundary conditions. The following proposition ensures the existence of at least one steady state equilibrium (with $n \geq 1$, and $l = 0$ or $l = 1$) for a high enough productivity h in human capital formation and any configuration of all the other parameter values, provided $\delta v^{1-\varepsilon} < 1$ (which implies either intertemporal complementarity, with $\varepsilon > 1$, or the possibility of a significant sibship size effect in the case of intertemporal substitutability, with $\varepsilon < 1$).

PROPOSITION 3. *Assume that* $\delta v^{1-\varepsilon} < 1$. *Then there exists a steady state equilibrium with* $n \geq 1$ *for a high enough value of the productivity parameter* h *in human capital formation.*

Proof. Refer to Figure 1. By continuity, a steady state equilibrium with $n \geq 1$ exists if the graph of $hF(n)$ is (i) above or coinciding with $G(0)$ at $n = 1$, and (ii) below $G(1)$ for n close enough to v. Condition (i) can be written as

$$hF(1) = h \frac{\delta(v-1)(1-\varepsilon)/\alpha - (1-\delta)}{v} \frac{}{\delta(1-\varepsilon)/\alpha + 1 - \delta} \geq \frac{\sigma}{w} = G(0). \qquad \textbf{(15)}$$

As $\nu > 2$ and $(1 - \varepsilon)/\alpha \geq 1$ by assumption, $F(1) > 0$. Hence, the inequality is satisfied for h large enough. Condition (ii) can be written as

$$hF(\nu) = -h\frac{(1/\nu)\,\delta\nu^{-\varepsilon}\left(1 - \delta\nu^{1-\varepsilon}\right)}{\delta\nu^{1-\varepsilon}\left(1 - \varepsilon\right)/\alpha + 1 - \delta\nu^{1-\varepsilon}} < \frac{\sigma/w - 1}{1 - 1/\lambda} = G(1). \qquad (16)$$

As $\delta\nu^{1-\varepsilon} < 1$ by assumption, $F(\nu) < 0$, so that the inequality is again satisfied for h large enough. ∎

Notice that the assumption $\delta\nu^{1-\varepsilon} < 1$ of Proposition 3 implies the usual boundary condition that benefit is greater than cost for a low quantity and smaller for a high one. Under this condition, we generically obtain existence of an uneven number (typically one or three) of steady state equilibria. This boundary condition is, however, not necessary for existence of a steady state equilibrium.

4.3. The Sibship Size Effect and the Demographic Transition

Our model has not been designed to analyze the demographic transition. Some variables that are crucial for that purpose, in particular those that are linked to age, such as schooling, retirement, and mortality [see, for instance, Boucekkine et al. (2002)], are ignored in our modeling, or at least only cursorily taken into account. Also, we are restricting our study to the analysis of steady states, without developing the corresponding transitional dynamics. In spite of this, some remarks related to the demographic transition can be made even in this simplified framework.

We have shown the possible multiplicity of steady states, a possibility that opens the way to transitions between them, for instance from the full child labor regime with high fertility and low family consumption to the full schooling regime with low fertility and high family consumption. "Multiple steady states mean that history and luck are critical determinants of a country's growth experience" [Becker et al. (1990, p. S14)].

An alternative approach consists in assuming that some exogenous shock modifies parameter values, destroying a high-fertility steady state equilibrium and pushing the economy into a subsisting, now possibly unique, low-fertility one. Progress in education facilities, emphasized by Boucekkine et al. (2007), has certainly increased educational attainment, which translates in our model into an increase in the productivity parameter h and possibly also into an increase in the sensitivity $1/\lambda$ to the time actually spent in the education system. It may also have contributed to greater autonomy of the process of human capital formation relative to the family circle, possibly moderating the sibship size effect, which translates into a decrease of $1/\nu$. Such moderation of the sibship size effect may in addition result from improvements in public health, of which children were the foremost beneficiaries. We thus have three potential shocks with different effects, the resultant of which can only be established by simulation.

We may, however, notice that a higher ratio h/λ implies, by (10), a higher value of $\widehat{n}$, shifting the vertical segment of the cost curve $G(n)$ to the right without displacing, by (12), the intersection of the benefit curve $hF(n)$ with the horizontal axis. This, together with the higher slope of this curve associated with a larger h, may well result in a shift of the point $hF(\widehat{n})$ below the step $G(1)$, destroying any steady state equilibrium with child labor and leaving the full-schooling steady state as the unique steady state equilibrium. In other words, increasing the value of the productivity parameter h entails the existence of (possibly multiple) steady state equilibria, but it eventually ensures uniqueness, under the full-schooling regime. With very high productivity in human capital formation, the benefit of an extra child decreases too fast with the number of children for the sibship size effect to build up and sustain a regime with child labor and high fertility. This is made explicit in the following proposition.

PROPOSITION 4. *Assume that* $\delta v^{1-\varepsilon} < 1$. *Then* full schooling *is the only possible steady state regime for a high enough value of the productivity parameter h in human capital formation: child labor is excluded in steady state equilibrium.*

Proof. Refer again to Figure 1. By Proposition 3, a high enough value of h leads to the inequalities $hF(1) \geq G(0)$ and $hF(v) < G(1)$, which together imply the existence of steady state equilibria, multiplicity not being excluded. However, if h is high enough for $hF(\widehat{n}) < G(1)$ to obtain with $\widehat{n}$ solving equation (10), a stronger condition than $hF(v) < G(1)$, steady state equilibria with child labor are ruled out if F is decreasing for $n > \widehat{n}$, and an equilibrium with full schooling is alone possible. Consider indeed the function

$$F(n) = \frac{(1/n - 1/v)(1-\varepsilon)/\alpha - (1/v)\left(1 - \delta n^{1-\varepsilon}\right)}{(1-\varepsilon)/\alpha - 1 + \delta^{-1}n^{\varepsilon-1}} \equiv \frac{\mathcal{N}(n)}{\mathcal{D}(n)}, \qquad (17)$$

as defined by (12). Distinguish the two cases of intertemporal complementarity and intertemporal substitutability. In the first case, with $\varepsilon > 1$, $\mathcal{N}$ is decreasing and $\mathcal{D}$ is increasing, so that F is decreasing overall. Thus, we must have $1 < \widehat{n}$, so that $hF(\widehat{n}) < G(1) < G(0) \leq hF(1)$. By (10), this is true if $h > \lambda/\delta(1 - 1/v)$, as already mentioned. In the case of intertemporal substitutability, with $\varepsilon < 1$, $\mathcal{D}'(n) < 0$ and

$$\mathcal{N}'(n) = -\frac{1-\varepsilon}{\alpha}n^{-2}\left(1 - \alpha\frac{n}{v}\delta n^{1-\varepsilon}\right) < 0, \qquad (18)$$

because α, n/v, and $\delta n^{1-\varepsilon}$ all belong to the interval $(0, 1)$ if $n < v$ and given the assumption that $\delta v^{1-\varepsilon} < 1$. This implies that F changes its sign only once, from positive to negative, as n increases from 0 to v. Moreover, the sign of $F'(n)$ is equal to the sign of $\mathcal{N}'(n)\mathcal{D}(n) - \mathcal{N}(n)\mathcal{D}'(n)$, which is negative if $\mathcal{N}(n)$ [and hence $F(n)$] is negative. So the function F, once negative, is always decreasing.
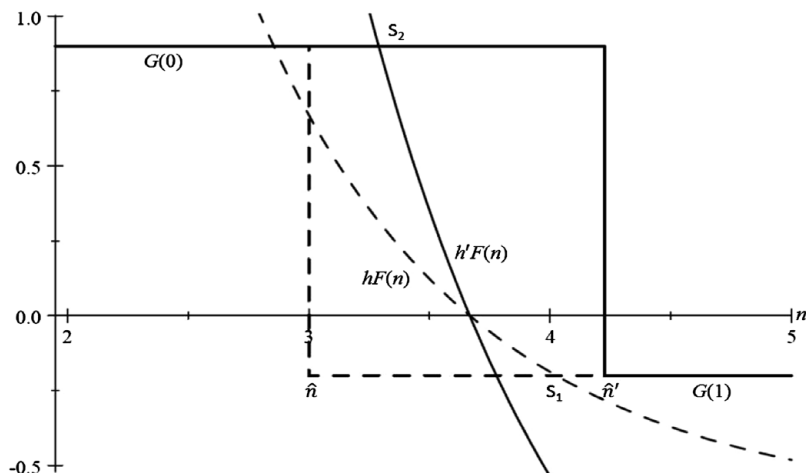
FIGURE 2. Illustration of the demographic transition.

As a consequence, there is no steady state at $n \geq \widehat{n}$ if

$$
hF\left(\widehat{n}\right) = \frac{\lambda\delta^{-1}\widehat{n}^{\varepsilon-1}\left(1-\varepsilon\right)/\alpha - \left(h/\nu\right)\left(1-\delta\widehat{n}^{1-\varepsilon}\right)}{\left(1-\varepsilon\right)/\alpha - 1 + \delta^{-1}\widehat{n}^{\varepsilon-1}} < \frac{\sigma/w-1}{1-1/\lambda} = G\left(1\right),
$$

$$\textbf{(19)}$$

using (10). This inequality is satisfied for a high enough value of h, provided $\delta\widehat{n}^{1-\varepsilon}$ (which depends upon h) is kept away from 1. If $\varepsilon > 1$, $\delta\widehat{n}^{1-\varepsilon} < 1$ results from $\widehat{n} > 1$, which has already been imposed. If $\varepsilon < 1$, it results from $\widehat{n} < \nu$, as we have assumed $\delta\nu^{1-\varepsilon} < 1$. ∎

Proposition 4 suggests an illustration of the demographic transition in our simplified framework.

Figure 2 illustrates one possible outcome of such shocks. For simplicity, we ignore in this example any shock on λ or ν. The situation depicted in Figure 1 is reproduced here by the dashed curves. The solid curves correspond to the same configuration of parameter values, except for a higher value of the productivity parameter h.[15] By increasing $\widehat{n}$ from 3 to approximately 4.22, and by pivoting the graph of $hF(n)$ clockwise around the point where it cuts the n-axis, the increase in h destroys the steady state equilibrium S_1 with full child labor (as well as the one with partial child labor) and makes the economy switch to the unique remaining steady state S_2 with full schooling, less fertility, and higher family income.

5. CONCLUSION

Intergenerational transfers are part of a long-running debate on child labor and standards of living. In this debate, some believe that during the industrial revolution

child labor was not an economic necessity, and therefore the standard assumption of downstream intergenerational transfers is perfectly appropriate. However, others view child labor as an economic necessity imposed by poverty, so that the standard assumption cannot apply.

As soon as we weaken the assumption on intergenerational transfers, and allow child labor to generate a positive net income, we have difficulty in theorizing endogenous fertility on the basis of the Beckerian trade-off between children's quantity and quality. We have shown that the introduction of a negative sibship size effect in child's human capital formation, and hence ultimately in the parent's utility function, is a possible answer to this difficulty, because it restores the trade-off. As long as that effect remains weak, a situation that seems appropriate in developed economies, the model works as under the usual assumptions that the sibship size does not essentially affect the siblings' future income and that intergenerational transfers are necessarily downstream. In poor economies, the effect may, however, be strong enough to reverse the sense of intergenerational transfers.

Allowing the intensity of the effect to increase with sibship size favors the emergence of multiple steady states in a dynamic setting, with contrasting regimes of child labor, high fertility, low income, and transfers from children to parents vs. child schooling, low fertility, high income, and transfers from parents to children. This kind of equilibrium multiplicity has been very much present in recent models designed to analyze the demographic transition, where the multiplicity can indeed be ascribed to the sibship size effect. However, as this effect was generally introduced in the silent form of the dilution of parents' time required to raise their children, its significance remained essentially unnoticed.

Multiplicity of steady states and the possibility of switching between contrasting regimes according to "history and luck" is one possible approach to the demographic transition. In this context, however, multiplicity itself may be less relevant than the possibility of associating distant steady states with close configurations of parameter values. We have shown, in our simple framework, that a continuous increase in the productivity of human capital formation can destroy the steady state with high fertility and low income, and consequently move the economy to the sole remaining steady state with low fertility and high income, provided the intensity of the sibship size effect is sufficiently sensitive to variations of the sibship size.

NOTES

1. The first paper on this theme [Becker (1960)] gave rise to a large demography-oriented literature [Becker and Lewis (1974), Becker and Tomes (1976), Becker (1981), and many others], which eventually shifted to questions of economic growth [see Galor and Weil (2000)].

2. See Caldwell (1981), Nardinelli (1990), Schellekens (1993), and Horrell and Humphries (1997) for the historical perspective, and Dasgupta (1995) and Edmonds and Pavcnik (2005) for contemporaneous evidence.

3. See also Birdsall (1983), Psacharopulos and Arriagada (1989), and Knodel et al. (1990).

4. See King (1987), Downey et al. (1999), Guo and VanWey (1999), Phillips (1999) and Downey (2001). The idea of resource dilution is also present, although not emphasized as such, in a series of economics papers, beginning with Becker et al. (1990) and published principally during the last decade. We will come back to these papers in the next section.

5. We should emphasize that not all poor countries or poor households have intergenerational transfers from children to parents. There is a debate in the literature on whether parents can survive without child labor, and whether net transfers to children are positive. On one hand, Basu and Van (1998) claim that child labor is a necessity, and that parents make use of it only because they have no other means of survival. In their own terms, "children's leisure or, more precisely, non-work is a luxury good in the household consumption" [Basu and Van (1998, p. 415)], a statement described as the "luxury axiom." On the other hand, some authors claim that this is not the case, and that child labor is used even when superfluous, a situation that, contrary to the preceding one, is of course compatible with downstream intergenerational transfers [see Edmonds and Pavcnik (2005) for a summary on this literature].

6. We are not aware of models explicitly introducing a sibship size effect in this context, although, as explained in note 4, resource dilution (a weak and implicit form of the sibship size effect) is present in a series of recent economic papers related to ours.

7. Jones and Schoonbroodt (2010) base their analysis of the demographic transition on the adoption of a dynastic model with intertemporal *complementarity* ($\alpha < 0$), where *the quantity and the quality* (the number and the well-being) of children *are substitutes* (instead of complements, for $\alpha > 0$). This allows fertility to respond negatively to decreased mortality and increased productivity growth, and hence to reproduce the observed pattern of the demographic transition.

8. The models referred to in the following do not necessarily adopt a dynastic specification (or its recurrent equivalent). This is, however, immaterial to the point we are discussing. The elasticity of the value function V in equation (5) has simply to be replaced by the elasticity of some other function representing each child's utility, leading to the same result. For instance, Baland and Robinson (2000), using a static model, introduce such a function W_c, depending only on child consumption. They ignore any sibship size effect ($\epsilon_{Hn}(l, n) \equiv 0$ in our notation), so that intergenerational transfers are necessarily downstream in their model.

9. The parent of generation $t - 1$ supplies l_t^P units of her time endowment in the labor market, and devotes the remaining time $(1 - l_t^P)$ to the education of her children. Each one of them, in a sibship of size n_t, will dispose one period ahead of $\tilde{H}\left((1 - l_t^P)/n_t\right)$ efficient units of labor, where $\tilde{H}$ is an increasing function. Hence, there is dilution of the time resource, leading to a negative effect of sibship size on human capital formation. See also Tamura (1994), where there is human capital dilution: each child's human capital is proportional to the parent's human capital divided by the number of children. As the parent's resources include, in addition to human capital, a goods endowment, there is also dilution of this endowment as the sibship size increases.

10. The same kind of results are also obtained in a model by Basu and Van (1998) where fertility remains exogenous, with sibship size having a wage-depressing effect in a context of general equilibrium.

11. To illustrate, consider the last five cited papers. In order to eliminate the phenomenon of the parent's time endowment dilution, we take $z = 0$ in Hazan and Berdugo (2002, p. 814); $s = 0$ in Blackburn and Cipriani (2005, p. 197 *in fine*), and Sugawara (2010, equation (3)); $\tau = 0$ in Chakraborty and Das (2005, p. 274 *in fine*); and $q = 0$ in Varvarigos and Zakaria (2013, equation (2)). The switch values of human capital that separate the different regimes then become infinite, namely $\tilde{h}$ in equation (12a) of Hazan and Berdugo; z^c in Proposition 3 and $\left(z_1^c, z_2^c\right)$ in Proposition 7 of Blackburn and Cipriani; $\underline{e}$ in equation (4) of Chakraborty and Das; $\tilde{h}$ in equation (12), $\hat{h}$ in equation (16), and $\overline{h}$ in equation (20) of Sugawara. In Varvarigos and Zakaria, where the regime switches are not explicitly detailed, the equilibrium number of children becomes infinite (equation (13)). A single regime remains, the one with maximum possible transfers from children to parents: no bequests in Blackburn and Cipriani; child labor and fertility at their upper extreme values in Hazan and Berdugo, Chakraborty and Das, Sugawara, and Varvarigos and Zakaria.

12. The initial assumption imposing the same sign on α and $1 - \varepsilon$ [hence $(1 - \varepsilon)/\alpha > 0$] is now reinforced in order to ensure concavity of the objective function with respect to n_t.

13. The condition $\delta v^{1-\varepsilon} < 1$ in the intertemporal substitutability case can be interpreted as the possibility of attaining a strong sibship size effect for a value of n compatible with finiteness of the objective function in the dynastic program.

14. The two curves are computed according to the following parameter values: $\alpha = -0.99$, $\delta = 0.3$, $\varepsilon = 2$, $\lambda = 2$, $v = 7$, $h = 105$, and $\sigma/w = 0.9$. The steady state with full schooling ($l = 0$) has low fertility ($n = 2.855$), high human capital [$H(0, 2.855) = 62.175$], and high family consumption in wage units ($c/w = 59.606$). These characteristics are reversed in the steady state with full child labor ($l = 1$): $n = 4.025$, $H(1, 4.025) = 22.31$, and $c/w = 22.715$. In the intermediate regime, we have $l = 0.346$, $n = \widehat{n} = 3$, $H(0.346, 3) = 49.607$, $c/w = 47.958$, and transfers from parents to children.

15. The parameter h is equal, as before, to 105 for the dashed curves and to 300 for the solid curves.

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APPENDIX: SECOND-ORDER CONDITION RELATIVE TO N_t

By differentiating the LHS of equation (11), we obtain the second derivative with respect to N_t of the objective function in program (2):

$$
\begin{aligned}
&-\delta^{t+1} \frac{N_{t+1}^{2-\varepsilon}}{N_t^3} c_{t+1}^{\alpha-1} \frac{wh}{\nu} \left(1 - \frac{l_t}{\lambda}\right) \left(2 + (1-\alpha) \frac{wh}{\nu} \left(1 - \frac{l_t}{\lambda}\right) \frac{N_{t+1}}{N_t c_{t+1}}\right) \\
&- (1-\alpha)\, \delta^{t-1} N_{t-1}^{-(1+\varepsilon)} c_{t-1}^{\alpha-2} \left(\sigma - wl_{t-1}+b_{t-1}\right)^2 \\
&- (1-\alpha)\, \delta^{t} N_{t}^{-(1+\varepsilon)} c_{t}^{\alpha-2} \left(\left(\sigma - wl_t +b_t\right)\frac{N_{t+1}}{N_t} - \frac{wh}{\nu}\left(1 - \frac{l_{t-1}}{\lambda}\right)\frac{N_t}{N_{t-1}} + \frac{\varepsilon c_t}{1-\alpha}\right)^2 \\
&- 2\delta^{t} \frac{N_t^{-\varepsilon}}{N_{t-1}} c_{t}^{\alpha-1} \frac{wh}{\nu}\left(1 - \frac{l_{t-1}}{\lambda}\right) - \left(\frac{1-\varepsilon}{\alpha} - 1\right)\frac{\varepsilon}{1-\alpha} \delta^{t} N_t^{-(1+\varepsilon)} c_t^{\alpha},
\end{aligned}
$$

which should be negative. All the terms of this sum but the last one [nonpositive under the assumption that $(1-\varepsilon)/\alpha \geq 1$] are indeed negative.